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Wissen lockt. Seit 1456

The NTF format: Extension of the logic specification

Melanie Taprogge, University of Greifwald
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jww. A. Steen, G. Sutcliffe, T. Scholl, C. Benzmüller, D. Fuenmayor

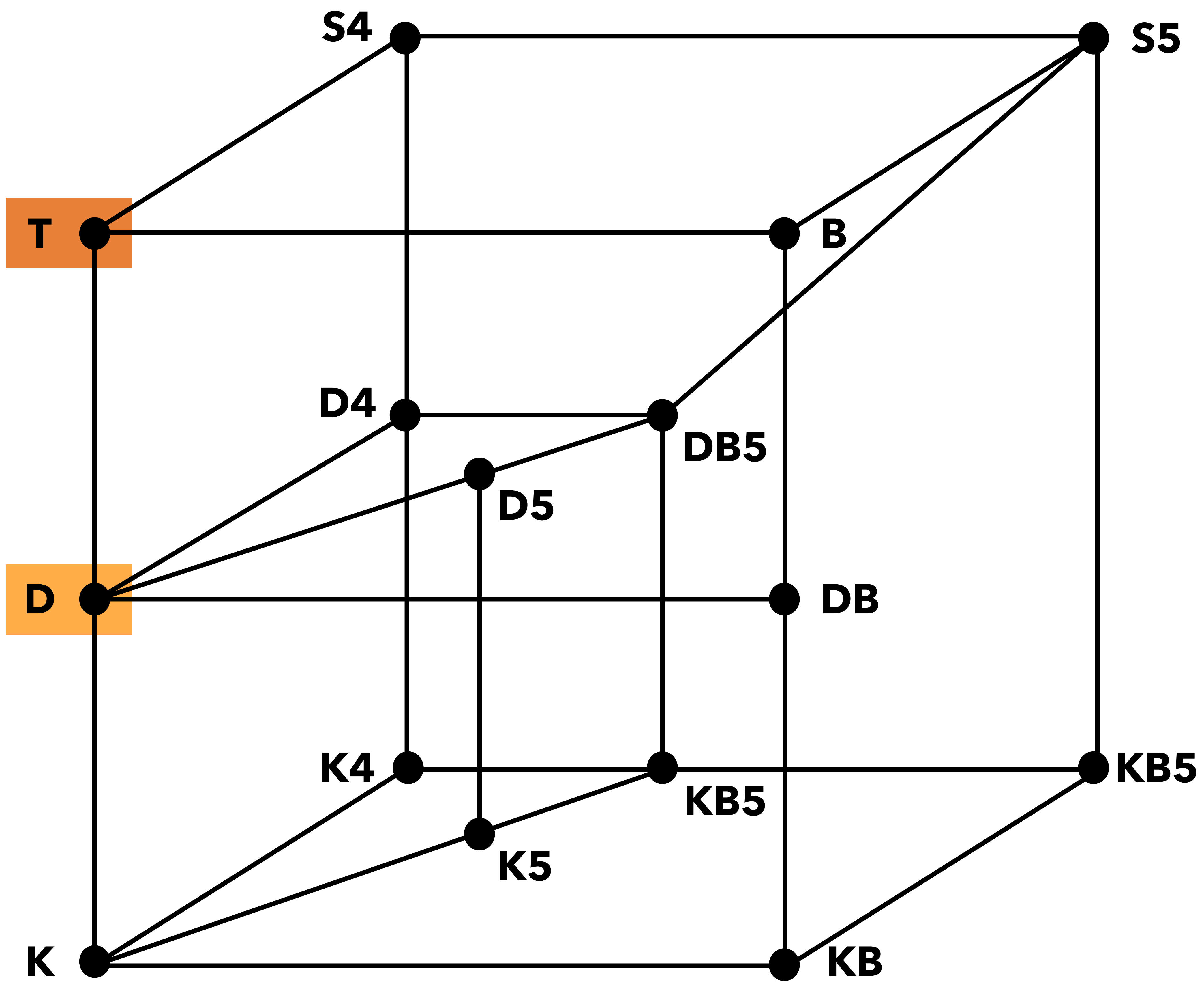
Characterisation of modal logics

We have seen that we can characterise modal logics based on the properties of the box operators …

Frame Properties	Axiom Schemes
Serial	$(D)\quad \Box A \supset \Diamond A$
Reflexive	$(T)\quad \Box A \supset A$
Transitive	$(4)\quad \Box A \supset \Box\Box A$
Euclidean	$(5)\quad \Diamond A \supset \Box\Diamond A$
Symmetric	$(B)\quad A \supset \Box\Diamond A$

… and that we can use the logics of the modal logic cube to define logics in the logic specification …

```
thf(logic_spec, logic, $modal == [
  . . .
      $modalities == [
          $modal_system_T,
          {$box(#1)} ==
              [$modal_system_D],
          . . .]]).
```



… but is that all there is to modal logics?

No!

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Irreflexive
$\neg xRx$

Frame Properties

Axiom Schemes

McKinsey Axiom
$\Box\Diamond A \supset \Diamond\Box A$

Interactions

$\Box_1 A \supset \Box_2 A$

How can we extend the TPTP syntax to account for this?

We can express axiom-schemes and frame properties in the existing syntax…

Frame Properties

-> Formulation of semantics in meta logic (HOL)

type of worlds

```
![X: $ki_world] :
    (~$ki_accessible(X,X))
```

Predicate representing *R*

Axiom Schemes

```
{$box} @ ({$dia} @ (A))
    => {$dia} @ ({$box} @ (A))
```

```
{$box(#1)} @ (A)
    => {$box(#2)} @ (A)
```

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How can we extend the TPTP syntax to account for this?

We can express axiom-schemes and frame properties in the existing syntax and include them in the logic specification!

Frame Properties

-> Formulation of semantics in meta logic (HOL)

type of worlds

```
![X: $ki_world] :
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Predicate representing *R*

Axiom Schemes

```
{$box} @ ({$dia} @ (A))
    => {$dia} @ ({$box} @ (A))
```

```
{$box(#1)} @ (A)
    => {$box(#2)} @ (A)
```

```
thf(logic_spec, logic, $modal == [
  $designation == $rigid,
  $domains == $constant,
      $modalities == [
          $modal_system_T,
          {$box(#1)} ==
              [$modal_system_D
               ![X: $ki_world] :
                (~$ki_accessible(X,X)),
               {$box} @ ({$dia} @ (A))
                => {$dia} @ ({$box} @ (A))],
          . . .
          {$box(#1)} @ (A)
            => {$box(#2)} @ (A),
          . . .
          ]]).
```

Summary

- The TPTP-Syntax was extended to allowed for the representation of FOML setups characterised by arbitrary **frame properties**, **axiom schemes** and **interactions**
- The implementation of an embedding of such setups into HOL can be used with ATP systems to reason within these non-trivial logics (implemented in LET, Leo-III)
- Encoding problems including interactions has posed a problem
- One example is the (simplified) Yale Shooting Problem [Baldoni 1998]

Logic definition:

$T: \quad \Box_{always} A \supset A$

$4: \quad \Box_{always} A \supset \Box_{always} \Box_{always} A$

$B_1: \quad \Box_{always} A \supset \Box_{load} A$

$B_2: \quad \Box_{always} A \supset \Box_{shoot} A$

Reasoning problem:

$1: \quad \Box_{always} \Box_{load}\, loaded$

$2: \quad \Box_{always} \left(loaded \supset \Box_{shoot} \neg alive\right)$

$C: \quad \Box_{load} \Box_{shoot} \neg alive$

Attempt at including B_1 as regular axioms in the QLMTP:
[Raths, Otten, 2012]

$\Box_{always}\, loaded \supset \Box_{load}\, loaded$

$\Box_{always} \neg loaded \supset \Box_{load} \neg loaded$

$\Box_{always}\, alive \supset \Box_{load}\, alive$

$\Box_{always} \neg alive \supset \Box_{load} \neg alive$

➡ Not provable!

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$2: \Box_{always} \left(loaded \supset \Box_{shoot} \neg alive \right)$

$C: \Box_{load} \Box_{shoot} \neg alive$

```
tff(modal_system, logic,
    $modal ==
      [ $modalities == [
          {$box(#always)} ==
            [$modal_axiom_T, $modal_axiom_4],
          {$box(#load)} ==
             $modal_system_K,
          {$box(#shoot)} ==
             $modal_system_K,
        {$box(#always)} @ (P)
          => {$box(#load)} @ (P),
        {$box(#always)} @ (P)
          => {$box(#shoot)} @ (P) ] ] ).
```

➡ Provable!

- This has (up to our knowledge) not been possible in any existing ATP systems before and yielded the first provable version of the shown problem.