



# The NTF format for non-classical logics

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## TPTP is the standard in (classical) theorem proving

**Our goal:** 'gracefully' extend to non-classical logics

- ▶ Minimal syntactic changes
- ▶ Uniform syntax for all non-classical logics
- ▶ Consistency throughout TPTP dialects
- ▶ User-friendly syntax  
*(easy reading and writing of problems)*
- ▶ Developer-friendly syntax  
*(easy parsing, minimal no. of cases to consider)*



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**Typed first-order logic (TXF):** Recap on TPTP syntax

```
tff(dog_decl,type, dog: $tType ).
tff(human_decl,type, human: $tType ).
tff(owner_of_decl,type, owner_of: dog > human ).
tff(bit_decl,type, bit: (dog * human * $int) > $o ).
tff(hates_decl,type, hates: (human * human) > $o ).

tff(hate_the_multi_biter_dog,axiom,
  ! [D: dog,H: human,N: $int] :
    ( ( H != owner_of(D) & bit(D,H,N) & $greater(N,1) )
      => hates(H,owner_of(D)) ) ).
```

**Higher-order logic (THF):** Recap on TPTP syntax

```
thf(dog_decl,type, dog: $tType ).
thf(human_decl,type, human: $tType ).
thf(owner_of_decl,type, owner_of: dog > human ).
thf(owns_decl,type, owns: human > dog > $o ).

thf(owns_defn,definition,
  ( owns = ( ^ [H: human,D: dog] : ( H = ( owner_of @ D ) ) ) ) ).

thf(hate_the_multi_biter_dog,axiom,
  ! [Huddle: dog > $o]: ?[Group: human > $o]:
  ![D: dog]: ? [H: human]:
  ( (Huddle @ D) & (Group @ H) & (owns @ H @ D) ) ).
```



## Extend languages with new operator:

- ▶ New kind of connective:

| { connective\_name }

- ▶ connective\_name is either TPTP-defined:

e.g. { \$necessary }, { \$possible }, { \$knows }, ...

- ▶ or connective\_name is system-defined:

e.g. { \$\$future }, { \$\$obligation }, { \$\$permission }, ...

## Resulting language: NTF (non-classical typed form)

- ▶ Non-classical typed extended first-order form (NXF)

- ▶ first-order-like application style:

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  - ▶ canonical higher-order application style (curried):

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## Example in NXF:

```
tff(possible_dog_bit_owner,axiom,  
    {$dia} @ (? [D: dog] : bit(D,owner_of(D),1)) ).
```

```
tff(jon_says_necessary_truth,axiom,  
    ! [S: $o] : ( says(jon,S) => {$box} @ (S) ) ).
```

## Example in NHF:

```
thf(possible_jon_owns_biter,axiom,  
    ! [D: dog] :  
    ( ( bit @ D @ jon @ 1 )  
    => ( {$dia} @ ( owns @ jon @ D ) ) ) ).
```

```
thf(jon_says_he_must_feed_odie,axiom,  
    says @ jon @ ({$box} @ (feeds @ jon @ odie)) ).
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### **Optional parameters:** Every NCL connective may be parameterized

- ▶ For logics with families of operators, e.g. ...
  - ▶ multi-modal logics:  $\Box_i$
  - ▶ term-modal logics:  $[t]\phi$
  - ▶ propositional dynamic logic:  $[p \cup q]\phi$
  - ▶ epistemic logic:  $K_A\phi$ ,  $C_{\{A,B,C\}}\phi$ , ...

### **Representation:** key-value arguments

```
{ connective_name(param1 := value1, param2 := value2, ...) }
```

- ▶ ... where the params are functors,
- ▶ and the values are arbitrary terms

Allow hashed (#ed) index value as first argument:

```
{ connective_name(#index, param1 := value1, param2 := value2, ...) }
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```
tff(alice_knows_its_possible_odie_bit_jon,axiom,  
  {$knows(#alice)} @ ({$dia} @ (bit(odie,jon,1)) ) ).
```

```
tff(jon_says_common_knowledge,axiom,  
  ! [S: $o] :  
    ( says(jon,S) => {$common($agents:=[alice,bob,claire])} @ (S) ) ).
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```
thf(alice_knows_jon_owns_a_dog,axiom,  
  {$knows(#alice)} @  
  ? [D: dog] : ( owns @ jon @ D ) ).
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thf(alice_and_bob_know_jon_might_lie,axiom,  
  ! [S: $o] :  
    ( (says @ jon @ S )  
      => {$common($agents:=[alice,bob])} @ ({$dia} @ ~ S) ) ).
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### **A note on the format**

- ▶ NTF is a result of different deliberate design decisions:
  - ▶ Minimal parser extensions if TXF (or THF) is already supported
  - ▶ Prolog parsing compability (long-standing principle)
  - ▶ Syntax should be general enough to cover many (complicated) NCLs
  - ▶ Distinction of object-language and meta-language components



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### Advantages:

(1) n-ary operators remain n-ary:  $\{\$box(\#i)\} @ (\phi i)$  for  $\square; \phi$

(2) operators always use the same number of arguments (also in case NCL has e.g. indexed and unindexed box)

(3) no typing issues (meta-level objects – like index  $i$  – may not be part of the term/formula language)



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  - ▶ Syntax should be general enough to cover many (complicated) NCLs
  - ▶ Distinction of object-language and meta-language components
- ▶ NTF clearly is (a bit) more complex than single-purpose languages
  - ▶ If we only need to support (indexed) modal operators, things are syntactically simpler
  - ▶ If we only need to support unary (binary) NCL operators, things are syntactically simpler
  - ▶ ...



## From which logic does the formula $p \vee q$ come from?

Classical logic? Wrong! I meant intuitionistic logic.

## From which logic does the formula $\Box\phi \rightarrow \phi$ come from?

Modal logic **K**? Wrong! I meant **S5** ...

- ▶ Formula syntax alone not enough to let ATP systems know which logic we're in
- ▶ Introduce: Logic specification

```
| tff(formula_name, logic, logic_name == [ properties ] ).
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where ...

- ▶ *logic\_name* is the name of the logic family,
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## **As a start: Focused on (quantified) modal logics**



## Examples

### Example formulas of modal logics

Mono-modal:

- ▶  $\Box \text{raining} \rightarrow \Diamond \text{raining}$
- ▶  $\forall P (\Diamond \text{rich}(P) \vee \Diamond \neg \text{rich}(P))$
- ▶  $\neg \Box (\exists X \text{rich}(X))$

Multi-modal:

- ▶  $\Box_a \text{raining} \rightarrow \Diamond_b \text{raining}$
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## Representation in TPTP

### Connectives (mono-modal)

- ▶ { \$box }
- ▶ { \$dia }

### Connectives (multi-modal)

- ▶ { \$box(#i) }
- ▶ { \$dia(#i) }

### Examples from above:

```
tff(1, axiom, { $box } @ (raining) => { $dia } @ (raining) ).  
tff(2, axiom, ![P]: ( { $dia } @ (rich(P)) | ~({ $dia } @ (rich(P))) ) ).  
tff(3, axiom, ~ { $box } @ ( ? [X]: rich(X) ) ).
```

We also offer user-friendly names: { \$necessary }, { \$possible }, { \$knows }, ...



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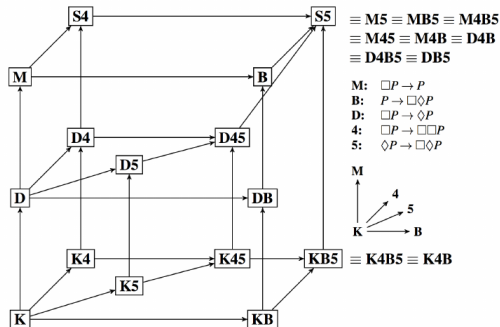
### Examples from above:

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We also offer user-friendly names: { \$necessary }, { \$possible }, { \$knows }, ...

## Modal logic: A family of many different logics

- ▶ Many parameters exist to create more specific modal logics
- ▶ Popular example: Properties of the box operator





1. **Axiomatization of  $\Box_i$**
2. **Quantification domains**
3. **Rigid/flexible designation of symbols**
4. **Term locality**



## 1. Axiomatization of $\Box_i$

- ▶ What properties does the box operators have?
- ▶ Depending on the application domain

Some popular axiom schemes:

Name	Axiom scheme	Condition on $R_i$	Corr. formula
K	$\Box_i(s \supset t) \supset (\Box_i s \supset \Box_i t)$	—	—
B	$s \supset \Box_i \Diamond_i s$	symmetric	$wR_i v \supset vR_i w$
D	$\Box_i s \supset \Diamond_i s$	serial	$\exists v. wR_i v$
T/M	$\Box_i s \supset s$	reflexive	$wR_i w$
4	$\Box_i s \supset \Box_i \Box_i s$	transitive	$(wR_i v \wedge vR_i u) \supset wR_i u$
5	$\Diamond_i s \supset \Box_i \Diamond_i s$	euclidean	$(wR_i v \wedge wR_i u) \supset vR_i u$
...	...	...	...

## 2. Quantification domains

## 3. Rigid/flexible designation of symbols

## 4. Term locality



## 1. Axiomatization of $\Box_i$

- ▶ What properties do the box operators have?

## 2. Quantification domains

- ▶ What is the meaning of  $\forall$ ?
- ▶ Several popular choices exist
  - (1) Varying domains: No restrictions
  - (2) Constant domains:  $\mathcal{D}_w = \mathcal{D}_v$  for all worlds  $w, v \in W$
  - (3) Cumulative domains:  $\mathcal{D}_w \subseteq \mathcal{D}_v$  whenever  $(w, v) \in R^i$
  - (4) Decreasing domains:  $\mathcal{D}_w \supseteq \mathcal{D}_v$  whenever  $(w, v) \in R^i$

## 3. Rigid/flexible designation of symbols

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## 1. Axiomatization of $\Box_i$

- ▶ What properties does the box operators have?

## 2. Quantification domains

- ▶ What is the meaning of  $\forall$ ?

## 3. Rigid/flexible designation of symbols

- ▶ Do all constants  $c \in \Sigma$  denote the same object at every world?
- ▶ Several popular choices exist

(1) Flexible constants:  $\mathcal{I}_w$  may vary for each world  $w$

(2) Rigid constants:  $\mathcal{I}_w(c) = \mathcal{I}_v(c)$   
for all worlds  $w, v \in W$  and all  $c \in \Sigma$

## 4. Term locality



## 1. Axiomatization of $\Box_i$

- ▶ What properties does the box operators have?

## 2. Quantification domains

- ▶ What is the meaning of  $\forall$ ?

## 3. Rigid/flexible designation of symbols

- ▶ Do all constants  $c \in \Sigma$  denote the same object at every world?

## 4. Term locality

- ▶ What is the domain for the interpretation of terms  $t$ ?
- ▶ At least two common possibilities:

(1) Local terms: Interpretation of  $t$  at world  $w$  is an element of  $\mathcal{D}_w$

(2) Global terms: Interpretation of  $t$  at world  $w$  is some element from  $\bigcup_{w \in W} \mathcal{D}_w$



1. **Axiomatization of  $\Box_i$** 
  - ▶ What properties does the box operators have?
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  - ▶ What is the meaning of  $\forall$ ?
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→ many different logics



### Use logic specification to encode specific logic

```
| tff(formula_name, logic, $modal == [ properties ] ).
```

- ▶ \$modalities for the properties of  $\Box_i$
- ▶ \$domains for the properties of  $\mathcal{D}_W$
- ▶ \$designation for the properties of  $\mathcal{I}_W$
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### Allowed values:



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## Allowed values:

**\$modalities:** \$modal\_system\_X or [\$modal\_axiom\_Y<sub>1</sub>, ..., \$modal\_axiom\_Y<sub>n</sub>] for ...  
X ∈ {K, KB, K4, K5, K45, KB5, ..., S4, S5},  
Y<sub>i</sub> ∈ {K, T, B, D, 4, 5, C, ...}.

**\$domains:** \$constant, \$varying, \$cumulative, \$decreasing

**\$designation:** \$rigid, \$flexible

**\$terms:** \$global, \$local



## Simple example:

```
tff(simple_spec,logic,  
    $modal == [  
        $designation == $rigid,  
        $domains      == [ $constant, some_user_type == $varying ],  
        $terms        == $global,  
        $modalities   == $modal_system_S5 ] ).
```

## More complex example:

```
tff(complex_spec,logic,  
    $modal == [  
        $designation == [ $flexible, sun == $rigid ],  
        $domains     == [ $constant,  
                          planet_type == $varying],  
        $terms       == $local,  
        $modalities  == [ $modal_system_K,  
                          {$box(#1)} == $modal_system_KB,  
                          {$box(#2)} == [ $modal_axiom_K,  
                                          $modal_axiom_4 ] ] ).
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```





## TPTP integration

- ▶ TPTP v9.0.0 contains 147 NTF (monomodal logic) problems: 132 NXF + 15 NHF
- ▶ TPTP4X utility for NTF
- ▶ proof verification via GDV
- ▶ AGMV model verifier
- ▶ IDV derivation viewer (for NTF)
- ▶ IIV interpretation viewer for Kripke models
- ▶ scala-tptp-parser package available

## Automation of modal logics

Existing translations to bridge to ...

- ▶ KSP
- ▶ nanoCoP-M
- ▶ MleanCoP
- ▶ any TFF/THF reasoner via Logic Embedding Tool



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## Possible (exotic?) future

### Dynamic epistemic logics (PAL)

- ▶ Does  $[\varphi!] \varphi$  hold?

tff(c1, conjecture,  $\{\$box(\$announce := \phi)\} @ (\phi) \}$  ).

- ▶ Does  $[\varphi!] (C_{a,b,c} \varphi)$  hold?

tff(c2, conjecture,  $\{\$box(\$announce := \phi)\} @ (\{common(\$agents := [a,b,c])\} @ (\phi)) \}$  ).

### Term modal logics

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tff(c4, conjecture,  $\{\$box(\$term := f(X))\} @ (\phi) \}$  ).

Of course, concrete syntax needs to be discussed.



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## Summary

- ▶ NTF: Generic NCL syntax extension of the TPTP
  - ▶ Current focus: Modal logic
- ▶ NTF problems in TPTP v9.0.0
- ▶ Solutions as usual in the TSTP
- ▶ Many TPTP tools and infrastructure extended to NTF
- ▶ More logics to come (with your help?)

Not discussed here:

- ▶ There exist means of automation for the presented logics
  - ▶ Based on shallow semantical embedding to HOL
- ▶ Recent experiments for quantified modal logics (QMLTP):  
Competitive performance wrt. native modal logic provers

