WALDMEISTER:

A Prover for Unit Equational Deduction

Bernd Löchner[1] Thomas Hillenbrand[2]

[1]Technische Universität Kaiserslautern
[2]Max-Planck-Institut für Informatik Saarbrücken

Unit Equational Logic

- Example: group axiomatization

$$\mathcal{E}: \; (x+y)+z = x+(y+z) \qquad x+0=x \qquad x+(-x)=0$$

  Word problem: Does $\mathcal{E} \models x = --x$ hold?

- Tackle word problem with Knuth-Bendix completion
  – idea: equations $l = r$ oriented into rewrite rules $l \to r$
  – aim: $\mathcal{E} \models s = t$ iff $s\downarrow \equiv t\downarrow$
  – price: saturation of rules necessary

- Perform fully automated proof search
  Return proof log in case of success

WALDMEISTER Searching for a Proof

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***********************************************************************
************************* COMPLETION - PROOF *************************
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new rule:                 1  +(x1,0) -> x1
new rule:                 2  +(x1,-(x1)) -> 0
new rule:                 3  +(+(x1,x2),x3) -> +(x1,+(x2,x3))
new rule:                 4  +(x1,+(0,x2)) -> +(x1,x2)
new rule:                 5  +(x1,-(0)) -> x1
new rule:                 6  +(x1,+(-(x1),x2)) -> +(0,x2)
new rule:                 7  +(0,-(-(x1))) -> x1
new rule:                 8  +(x1,-(-(x2))) -> +(x1,x2)
remove rule:              7
new rule:                 9  +(0,x1) -> x1
remove rule:              4
simplify rhs of rule:     6
new rule:                10  -(0) -> 0
remove rule:              5
new rule:                11  -(-(x1)) -> x1
remove rule:              8
joined goal:            1  c ?= -(-(c)) to c
        +---------------------------+
        |   this proves the goal    |
        +---------------------------+

Proved Goals:
No.  1:  c ?= -(-(c)) joined, current: c = c
1 goal was specified, which was proved.

Waldmeister states: Goal proved.
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WALDMEISTER Presenting a Proof

Consider the following set of axioms:

Axiom 1: $x + 0 = x$
Axiom 2: $x + (-x) = 0$
Axiom 3: $(x + y) + z = x + (y + z)$

This theorem holds true:

Theorem 1: $x = --x$

Proof:

Lemma 1: $0 + (--x) = x$

$0 + (--x)$
= by Axiom 2 RL
$(x + (-x)) + (--x)$
= by Axiom 3 LR
$x + ((-x) + (--x))$
= by Axiom 2 LR
$x + 0$
= by Axiom 1 LR
x

Lemma 2: $x + (--y) = x + y$

$x + (--y)$
= by Axiom 1 RL
$(x + 0) + (--y)$
= by Axiom 3 LR
$x + (0 + (--y))$
= by Lemma 1 LR
$x + y$

Lemma 3: $0 + x = x$

$0 + x$
= by Lemma 2 RL
$0 + (--x)$
= by Lemma 1 LR
x

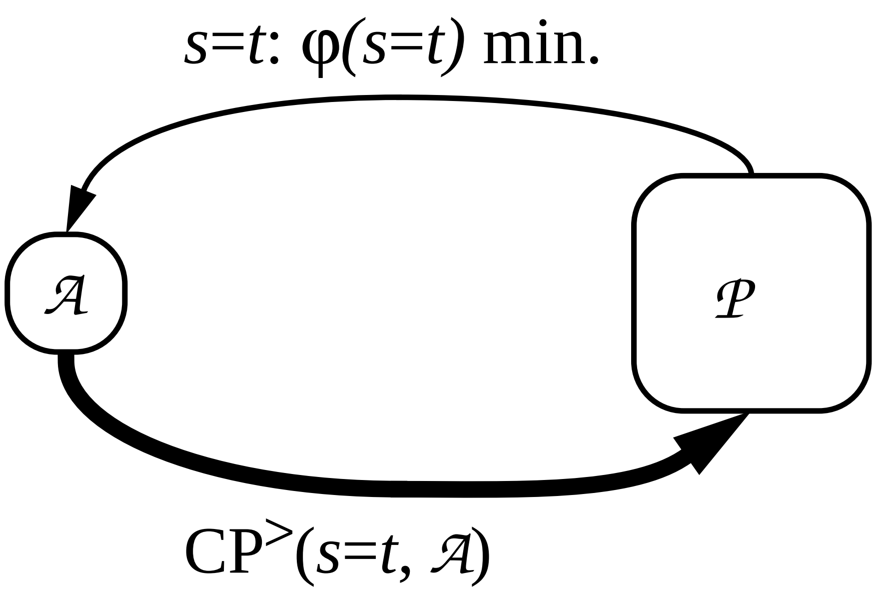
Theorem 1: $x = --x$

x
= by Lemma 3 RL
$0 + x$
= by Lemma 2 RL
$0 + (--x)$
= by Lemma 3 LR
$--x$

Key Features

- UEQ subdomain of particular characteristic
  no disjunctions $\rightsquigarrow$ no combinatorics on formula level
  redundancy notion centered around rewriting

- Carefully designed main loop & data structures engineering
  control parameters instantiated according to algebraic structure
  specialized redundancy criteria



WALDMEISTER in 2006

- Useful: cancellation rule added as simplification plus minor strategy rewrites

- Application scenarios:
  - mostly: educational; reference implementation
  - theorem proving in algebraic structures
  - computer linguistics (sic)

- Ongoing work on WALDMEISTER:
  - divide & conquer proof search?
  - continue integration into computer algebra system
  - exploration of new strategies

Web Page at waldmeister.org

Waldmeister - Theorem Prover http://www.mpi-sb.mpg.de/~hillen/waldmeister/

ABOUT WALDMEISTER | ALL TIME NEWS

As you might already know, Waldmeister (asperula odorata, woodruff) is an ingredient for a very popular potable. It is also liked as aroma for sodas. But no, the Waldmeister we are talking about here, you cannot use for your potion. Well, maybe you try and make such a potion. If you are a logic-wizzard after drinking from it, please contact us... Because the Waldmeister we are talking about here is a highly efficient theorem prover.

So be welcomed in the world of Waldmeister, which is the world of logical theorems.

Waldmeister is a theorem prover for unit equational logic. Its proof procedure is unfailing Knuth-Bendix completion [BDP89]. Waldmeister's main advantage is that efficiency has been reached in terms of time as well as of space. Within that scope, a complete proof object is constructed at run-time. Read more about the implementation.

ALL TIME NEWS

For his book

A New Kind Of Science

which is hot from the press, Stephan Wolfram has employed our system to carry out investigations in the area of singleton axiom systems for Boolean algebra. Pages