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 + external SMT procedure (currently: Yices or CVC3)
pass input and derived clauses to SMT, treat SMT refutations like SPASS inferences



Thanks to

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(OK, that doesn't leave much for me ...)



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 seem to be very hard for Z3 (due to peculiarities of the encoding?)

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- \rightsquigarrow Fixpoint problems in the TPTP SWW domain
 - seem to be very hard for Z3 (due to peculiarities of the encoding?)
 - SPASS+T finds proofs using superposition and chaining (inferences: \sim 10000, proof length: \sim 1000).

Second type of problems where SPASS+T has an edge:

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$$orall a.$$

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 \rightarrow
 $\exists j. read(a, j) = 50.$

But these are intuitively easy:

One superposition step + (trivial) variable elimination.

Why does Z3 fail here?

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 There are many techniques to solve arithmetic problems (DPLL(T), trigger-based instantiation, variable elimination, superposition, chaining, ...).

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- There is still a lot of room for improvements.
- By the way:

We need more interesting TFA problems in the TPTP.

CASC-23 TFA division: 96/100 problems solved.

SMT Refut	Std Axioms	Built-in Arithm	#Problems
+	+	+	2
+	+	_	0
+	_	+	1
+	_	_	15
_	+	+	26
_	+	_	4
_	_	+	48

Caveat: non-deterministic due to unsynchronized parallelism.